Question Paper with Final (Revised) Answer Key for the Post of	**Lecturer, Applied Science & Humanities (Polytechnic) Mathematics**	**held on 27-03-2021**

Itemcode : **HT1001**

Q1: Which one of the following is true?

A	Every first countable topological space is separable.
B	The product of two Hausdroff spaces may not be a Hausdroff space.
C	Every compact Hausdroff space is connected.
D	Every finite T_2 space is discrete.

Correct Ans: **D**

Itemcode : **HT1002**

Q2: Which one of the following is true?

A	A Banach space need not be closed.
B	A closed subspace of Banach space is complete.
C	In a Banach space, every absolutely convergent series need not be convergent.
D	The normed linear space $(\mathbb{R}^n, \|\|.\|\|)$ is not complete where $\|\|.\|\|$ is an arbitrary norm on $\mathbb{R}^n$.

Correct Ans: **B**

Itemcode : **HT1003**

Q3: The order of convergence of Newton Raphson method is

A	1
B	2
C	3
D	4

Correct Ans: **B**

Itemcode : **HT1004**

Q4: Let p(x) be a quadratic polynomial that takes the value y(1) = 1, y(2) = 2, y(3) = 10, then the value of y(4) by Newton's forward difference formula is

A	17
B	18
C	20
D	25

Correct Ans: **D**

Itemcode : **HT1005**

Q5: By using Euler's method, the value of y when x = 0.1, given that y(0) = 1 and $y' = xy + y^2$

A	1.01
B	1.1
C	1.2
D	1.3

Correct Ans: **B**

Itemcode : **HT1006**

Q6: For the points (1,8) and (3,9), the linear polynomial P(x) that pass through these two points is

A	$\frac{15}{2}+\frac{x}{2}$
B	$\frac{5}{2}+\frac{x}{2}$
C	$\frac{15}{2}-\frac{x}{2}$
D	$15-\frac{x}{3}$

Correct Ans: **A**

Itemcode : **HT1007**

Q7: If X is a continuous random variable whose probability distribution function is given by $f(x)=k(x-x^2), 0 \leq x \leq 1$, then $P\left(X>\frac{1}{2}\right)$ is

A	1/6
B	1/24
C	½
D	1

Correct Ans: **C**

Itemcode : **HT1008**

Q8: The residue of the function $f(z)=\frac{1+\cos(z)}{e^z-1}$ at $z=0$ is

A	1
B	2
C	-1
D	0

Correct Ans: **B**

Itemcode : **HT1009**

Q9: The radius of convergence of the series $\sum \frac{e^n}{n} z^n$ is

A	e
B	e^{-1}
C	∞
D	0

Correct Ans: **B**

Itemcode : **HT1010**

Q10: The function $f(z) = \frac{\cos\left(\frac{\pi z}{2}\right)}{\sin(\pi z)}$ has poles at

A	all odd integers
B	all even integers
C	all integers
D	all integers of the form 3k + 1, k an integer

Correct Ans: **B**

Itemcode : **HT1011**

Q11: If $|z-1| = |z-2|$, then

A	*Re(z)* = 0
B	*Re(z)* = 1
C	*Re(z)* = 1/2
D	*Re(z)* = 3/2

Correct Ans: **D**

Itemcode : **HT1012**

Q12: Let $f: \mathbb{D} \longrightarrow \mathbb{D}$ be a holomorphic function with $f(0) = 0$ where $\mathbb{D}$ is a unit disc $\{z: |z| < 1\}$. Then which one of the following is not correct?

A	$|f(1/4)| \leq 3/4$
B	$|f(3/4)| \leq 1/2$
C	$|f(1/4)| \leq 1/2$
D	$|f(1/2)| \leq 1/2$

Correct Ans: **B**

Itemcode : **HT1013**

Q13: The solution of initial value problem $y'' - 10y' + 21 = e^{3x}, \quad y(0) = y'(0) = 0$ is

A	$y = \frac{-1}{8}e^{3x} + \frac{1}{8}e^{7x} - \frac{x}{7}e^{3x}$
B	$y = \frac{-1}{16}e^{3x} + \frac{1}{16}e^{7x} - \frac{x}{4}e^{3x}$
C	$y = \frac{-1}{8}e^{3x} + \frac{1}{8}e^{7x} - \frac{x\text{^}2}{7}e^{3x}$
D	$y = \frac{-1}{16}e^{3x} + \frac{1}{16}e^{7x} - \frac{x\text{^}2}{7}e^{3x}$

Correct Ans: **B**

Itemcode : **HT1014**

Q14:

The Laplace transform of the function $f(t) = 8e^{3t}\sin(\sqrt{3}t) - e^{5t}$ is

A	$\frac{8(s-\sqrt{3})}{(s-\sqrt{3})^2+\sqrt{3}} - \frac{1}{s+5}$
B	$\frac{8(s-3)}{(s-3)^2+\sqrt{3}} - \frac{1}{s-5}$
C	$\frac{8(\sqrt{3})}{(s-3)^2+3} - \frac{1}{s-5}$
D	$\frac{8(s-\sqrt{3})}{(s-\sqrt{3})^2+3} - \frac{1}{s+5}$

Correct Ans: **C**

Itemcode : **HT1015**

Q15: Find the inverse Laplace transform of the function $F(s) = \frac{s}{(s^2+1)^2}$.

A	$cos(t)$
B	$2tsin(t)$
C	$(tsin(t))/2$
D	$(tcos(t))/2$

Correct Ans: **C**

Itemcode : **HT1016**

Q16: The solution of initial value problem $y' = 2 - t$, $y(0) = 1$ is

A	$y = 1 - 2t + \frac{t^2}{2}$
B	$y = 1 + \frac{t}{2} - t^2$
C	$y = 1 - \frac{t}{2} + t^2$
D	$y = 1 + 2t - \frac{t^2}{2}$

Correct Ans: **D**

Itemcode : **HT1017**

Q17: The total work done by the force $\vec{F} = z\hat{i} - x\hat{j} + y\hat{k}$ moving a particle from (0,0,0) to (1,2,4) along a line segment is

A	10
B	5
C	15
D	8

Correct Ans: **B**

Itemcode : **HT1018**

Q18: The directional derivative of $f(x,y) = xe^y$ at (1,0) in the direction of the vector $\hat{\imath} + \hat{\jmath}$ is

A	$\frac{1}{\sqrt{2}}$
B	2
C	$\sqrt{2}$
D	$2\sqrt{2}$

Correct Ans: **C**

Itemcode : **HT1019**

Q19: The area lying between the parabola y = $4x^2$ and the line y = x is

A	1/16
B	1/6
C	1/32
D	1/96

Correct Ans: **D**

Itemcode : **HT1020**

Q20: The value of $\iint_S F \,.\hat{n}\, dS$ where $|\vec{F} = x\hat{\imath} - y^2\hat{\jmath} + z\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ is

A	2
B	3
C	4
D	6

Correct Ans: **B**

Itemcode : **HT1021**

Q21: The value of $\lim_{x\to\infty} \frac{\left(1+\frac{1}{x}\right)^x - 1}{e}$ is

A	1/*e*
B	-1/*e*
C	(1/*e*) - 1
D	1 - (1/*e*)

Correct Ans: **D**

Itemcode : **HT1022**

Q22: The total number of one to one functions from {0,1} to {1,2,3,4,5} is

A	2^5
B	20
C	5!

D	10

Correct Ans: **B**

Itemcode : **HT1023**

Q23: The sum of the series is $1+\frac{1}{2!}+\frac{1}{2}+\frac{1}{3!}-\frac{1}{3}+\frac{1}{4!}+\frac{1}{4}+\cdots$ equals

A	e - log(2)
B	-e + log(2)
C	e - log(2) + 2
D	-e + log(2)- 2

Correct Ans: **A**

Itemcode : **HT1024**

Q24: The value of $\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=0}^{2n}k^2$ is

A	1/3
B	2/3
C	4/3
D	8/3

Correct Ans: **D**

Itemcode : **HT1025**

Q25: Let a, b be distinct real numbers, then the number of distinct real roots of the equation $(x-a)^5+(x-b)^5=0$ is

A	1
B	2
C	3
D	Depends on the value of a, b

Correct Ans: **A**

Itemcode : **HT1026**

Q26: Consider the function $f(x)=\cos(|x-3|)+|x^2-1|$, then what are the points at which the function f is not differentiable?

A	3
B	1, -1
C	Differentiable every where
D	No where differentiable

Correct Ans: **B**

Itemcode : **HT1027**

Q27: The value of improper integral $\int_0^\infty(3+x^2)e^{-x}dx$ is

A	5
B	

	3
C	-2
D	0

Correct Ans: **A**

Itemcode : **HT1028**

Q28: Consider the map $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f(x,y) = (x^3 + 4y, x^2 + 3x + y^4)$, then

A	The function f is not continuous at (0,0).
B	The function f is continuous at (0,0) but not differentiable at (0,0).
C	The function f is differentiable at (0,0) but the derivative $Df(0,0)$ is not invertible.
D	The function f is differentiable at (0,0) and the derivative $Df(0,0)$ is invertible.

Correct Ans: **D**

Itemcode : **HT1029**

Q29: Find the volume of the parallelopiped whose coterminus edges are represented by the vectors $\vec{a} = 3\hat{\imath} - 7\hat{\jmath} - 5\hat{k}, \vec{b} = \hat{\imath} + 4\hat{\jmath} - 7\hat{k}, \vec{c} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$ is

A	242
B	0
C	55
D	33

Correct Ans: **A**

Itemcode : **HT1030**

Q30: Which one of the following statements is true?

A	$A = \emptyset$ implies $P(A) = \emptyset$.
B	For every set A, there exists $f: A \to P(A)$, such that f is onto.
C	For every set A, there exists $f: A \to P(A)$, such that f is one – one.
D	$\mathbb{N}$ and $\mathbb{Z}$ are perfect sets.

Correct Ans: **C**

Itemcode : **HT1031**

Q31: Number of fixed points of the function $f(x)$ = $|\log_e|x| \,|$ + x - 5 are

A	4
B	6
C	3
D	2

Correct Ans: **A**

Itemcode : **HT1032**

Q32: Which of the following limit exists?

A	$\lim_{n\to 0} \sin\left(\frac{1}{n}\right)$
B	$\lim_{n\to 0} e^{\left(\frac{1}{n}\right)}$
C	$\lim_{x\to 0} f(x)$ *where* $f:[0,1]\to\mathbb{R}$ is given by $f(x)=\begin{cases} p\sin\left(\frac{1}{q}\right), & x=\frac{p}{q}\ \textit{in}\ [0,1] \\ x, & \textit{otherwise}\end{cases}$
D	$\lim_{x\to 0} \frac{\cos x}{x}$

Correct Ans: **C**

Itemcode : **HT1033**

Q33: Choose the incorrect statement.

A	There are uncountably many sequences converging to the limit point of a subset $S \subseteq \mathbb{R}$.
B	For monotonic functions, the set of points where limit does not exist in uncountable.
C	$\lim_{x\to\pi} \frac{\lvert\sin x\rvert}{\sin x}$ fails to exist.
D	$\lim_{x\to\infty} \frac{\sin x}{x} = 0.$

Correct Ans: **B**

Itemcode : **HT1034**

Q34: Choose the incorrect statement.

A	All sequences are continuous.
B	A function in continuous on an interval I, such that f(I) is a subset of the set of rational numbers, then f is constant.
C	$A = \{f:\mathbb{R}\to\mathbb{R}\text{ is continuous} \mid f\text{ vanishes only on set of rational numbers}\} \neq \emptyset.$
D	Continuity is sufficient condition for Intermediate Value Property, not necessary.

Correct Ans: **C**

Itemcode : **HT1035**

Q35: Which of the following statements is false?

A	$f(x) = \sin x^2,\ x\in\mathbb{R}$ is continuous, but not uniformly continuous.
B	$f(x) = e^x,\ x\in\mathbb{R}$ is continuous, but not uniformly continuous.
C	$f(x) = \lvert x\rvert,\ x\in\mathbb{R}$ is continuous, but not uniformly continuous.
D	Lipschitz condition implies uniform continuity.

Correct Ans: **C**

Itemcode : **HT1036**

Q36:	Choose the correct statement.
A	$f_n(x) = \frac{x}{1+nx}$, $x \in [0,\infty)$ is uniformly convergent.
B	$f_n(x) = x^n$, $x \in [0,1]$ is uniformly convergent.
C	$f_n(x) = \begin{cases} n^2 x, & 0 \le x < \frac{1}{n} \\ -n^2 x + 2n, & \frac{1}{n} \le x < \frac{2}{n} \\ 0, & \frac{2}{n} \le x \le 1 \end{cases}$ is uniformly convergent.
D	$f_n(x) = \frac{nx}{1+n^2 x}$, $x \in [0,\infty)$ is not uniformly convergent.
Correct Ans: **A**	

Itemcode : **HT1037**

Q37:	Which one of the following statements is false?
A	Number of linearly independent solutions for the system *Ax* = 0 is nullity $\eta(A)$.
B	If A is $n \times m$ matrix and b is $n \times 1$ vector, consider $Ax = b$, if rank $\rho(A{:}\,b) = m$. Then $Ax = b$ has at most one solution.
C	Existence of solutions of system of linear equations *Ax* = 0 depends upon rank $\rho(A)$.
D	Let $A_{n\times m} = [c_1, c_2, \ldots . c_m]$, $c_i \in F^n$. Then consider $Ax = b$, $S_1 = [c_1, c_2, \ldots . c_m, b]$. If S_1 is linearly dependent, then $Ax = b$ has a solution.
Correct Ans: **D**	

Itemcode : **HT1038**

Q38:	Pick the true statement.
A	$A_{3\times3}$ such that $A^3 = -I$. Then, A has 3 distinct eigenvalues
B	If V is a vector space over the field $\mathcal{F}$ with $\dim V = n$, then $T{:}\,V(\mathcal{F}) \to V(\mathcal{F})$ is diagonalizable implies that all the eigenvalues are distinct.
C	*T* is diagonalizable implies arithmetic and geometric multiplicities for each eigenvalue of *T* are equal.
D	The characteristic polynomial of T splits over $\mathcal{F}$ implies T is diagonalizable.
Correct Ans: **C**	

Itemcode : **HT1039**

Q39:	Which statement is false?
A	If $A_{n\times n}$ is an Idempotent matrix, $\rho(A) = Tr(A)$.
B	An Involutory matrix is diagonalizable over every field.
C	$a = (a_1, a_2, \ldots . . a_n)$, $b = (b_1, b_2, \ldots . . b_n) \in \mathcal{F}^n$, $A = a^T b$, $a \ne 0, b \ne 0$. Then A is diagonalizable iff$(A) \ne 0$.
D	Every odd skew-symmetric matrix is singular.

Correct Ans: **B**

Itemcode : **HT1040**

Q40: Pick the false statement.

A	$A_{n\times n}$ real matrix, then every eigen value of $A^T A$ is non-negative real.
B	$A \neq 0$ is real symmetric matrix, $Tr(A) = 0$. Then $q(x) = x^T Ax$ is indefinite.
C	$A_{n\times n}$ real symmetric non-singular matrix, such that $\exists\, x \in \mathbb{R}^n\ s.t.\, x^T Ax < 0$. Then $B = -A$ is positive-definite.
D	For A a real symmetric matrix, the range of $q(x) = x^T Ax$ is a subset of $\mathbb{R}^- \cup \{0\}$ for q negative-definite.

Correct Ans: **C**

Itemcode : **HT1041**

Q41: Which of the following is incorrect?

A	For $A_{n\times m}, B_{m\times n}$, $X = AB$, $Y = BA$. Then $\|X\| = \|Y\|$.
B	$T: V \to V$, $V = W_1 \oplus W_2$ where both W_1 and W_2 are T invariant subspaces. Then $m_T(x) = LCM\{m_{T_1}(x), m_{T_2}(x)\}$ where T_i are induced operators on W_i by T.
C	Let a matrix *A* has Jordon-canonical form, then the number of Jordon blocks corresponding to eigenvalue λ_i = Geometric multiplicity of λ_i.
D	In the Jordon-canonical form, the sum of the order of Jordon-blocks corresponding to eigenvalue λ_i = Arithmetic multiplicity of λ_i.

Correct Ans: **A**

Itemcode : **HT1042**

Q42: $\frac{dy}{dx} = ay - by^2; a, b > 0 \quad y(0) = y_0$. Then as $x \to \infty$, $y(x)$ tends to

A	0
B	$\frac{a}{b}$
C	$\frac{b}{a}$
D	y_0

Correct Ans: **B**

Itemcode : **HT1043**

Q43: Consider $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$; $a_0 \neq 0$; a_0, a_1, a_2 are continuous functions, y_1 and y_2 are 2 linearly independent solutions of the above equation. Which of the following statement is false?

A	The sets of zeroes of y_1 and y_2 are disjoint.
B	The set of zeroes is dense in $\mathbb{R}$.
C	The set of zeroes of y_1 is finite iff the set of zeroes of y_2 is finite.

D	If $\alpha_0 < \alpha_1$ for $\alpha_0,\ \alpha_1$ in the set of zeroes of y_1, then $\exists\ \beta$, a zero of y_2 such that $\alpha_0 < \beta < \alpha_1$.

Correct Ans: **B**

Itemcode : **HT1044**

Q44: The singular solution of $(xp - y)^2 = p^2 - 1$ is

A	$x^2 + y^2 = 1.$
B	$x^2 - y^2 = 1.$
C	$x^2 + 2y^2 = 1.$
D	$2x^2 + y^2 = 1.$

Correct Ans: **B**

Itemcode : **HT1045**

Q45: The general solution for $yp - xq = yz^2 e^{-(x^2+y^2)}$ is

A	$u = x^2 + y^2,\ \ v = x + \frac{1}{z} e^{(x^2+y^2)}.$
B	$u = x^2 + y^2,\ \ v = x - \frac{1}{z} e^{-(x^2+y^2)}.$
C	$u = x^2 - y^2,\ \ v = x + \frac{1}{z} e^{(x^2+y^2)}.$
D	$u = x^2 - y^2,\ \ v = x - \frac{1}{z} e^{-(x^2+y^2)}.$

Correct Ans: **A**

Itemcode : **HT1046**

Q46: The complete integral of $(p^2 + q^2)y - qz = 0$ is

A	$z^2 = a^2 y^2 + (ax + b)^2.$
B	$z^2 = ay^2 + (ax + b)^2.$
C	$z^2 = a^2 y + (ax - b)^2.$
D	$z = a^2 y^2 + (ax + b)^2.$

Correct Ans: **A**

Itemcode : **HT1047**

Q47: $(x-1)^2 u_{xx} - (y-2)^2 u_{yy} + 2xu_x + 2yu_y + 2xyu = 0$ is parabolic in $S \subseteq \mathbb{R}^2$ but not in $\mathbb{R}^2 \backslash S$. Then S is

A	$\{(x, y) \in \mathbb{R}^2 \colon x = 1 \text{ or } y = 2\}.$
B	$\{(x, y) \in \mathbb{R}^2 \colon x = 1 \text{ and } y = 2\}.$
C	

	$\{(x, y) \in \mathbb{R}^2 : x = 1\}.$
D	$\{(x, y) \in \mathbb{R}^2 : y = 2\}.$

Correct Ans: **A**

Itemcode : **HT1048**

Q48: Choose incorrect statement.

A	A group *G* is abelian if and only if every element in *G* is self-inverse.
B	*G* is an abelian group if and only if $(ab)^2 = a^2b^2 \; \forall a, b \in G.$
C	The identity element *e* is the only element with unit order.
D	Smallest non-abelian group of even order is a group of order 6.

Correct Ans: **A**

Itemcode : **HT1049**

Q49: Choose incorrect statement.

A	Number of generators of the group $\mathbb{Z}_{200}$ is 80.
B	$U(9)$ is a cyclic group.
C	Every subgroup other than Q_8 of Q_8 is abelian.
D	Every group can be expressed as the union of its two proper subgroups.

Correct Ans: **D**

Itemcode : **HT1050**

Q50: Choose the correct statement.

A	A group has an element of order *k* if and only if there is a subgroup of order *k*.
B	A group of order 20 can have two distinct subgroups of order 5.
C	A cycle in S_n can be expressed as a product of transpositions in infinitely many ways.
D	Number of 4-cycles in 4-symbols = 4.

Correct Ans: **C**

Itemcode : **HT1051**

Q51: Which one of the following statements is correct?

A	$\mathbb{Z}_2 \times \mathbb{Z}_4$ is cyclic.
B	Image of an abelian group under any group homomorphism is abelian.
C	Number of group homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{18}$ is 4
D	If *G* is a non-abelian group of order p^3 then the number of conjugate classes of *G* is given by $p^2 - p + 1$.

Correct Ans: **B**

Itemcode : **HT1052**

Q52: Which one of the following is incorrect?

A	$(\mathbb{C}^*, .)$ has an element of order n for every n.
B	

	$(\mathbb{C}^*,.)$ is cyclic.
C	If $x \in \mathbb{C}^*, o(x) < \infty \implies x = e^{i\theta}$.
D	$(\mathbb{R}^*,.)$ has exactly 2 elements of finite order.

Correct Ans: **B**

Itemcode : **HT1053**

Q53: Choose the incorrect statement.

A	Composition of group automorphisms is also an automorphism.
B	$Aut(K_4) \cong S_3$.
C	$o\big(SL(2,\mathbb{Z}_5)\big) = 100$.
D	$Aut(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2) \cong GL(3,\mathbb{Z}_2)$.

Correct Ans: **C**

Itemcode : **HT1054**

Q54: Choose the incorrect statement.

A	$\mathbb{Z}_7$ is a simple group.
B	Every group G with $\lvert G\rvert = p^3$ is simple.
C	Quotient group of an infinite group can be finite.
D	If G is non-abelian group of order p^3, then $\frac{G}{Z(G)} \cong \mathbb{Z}_p \times \mathbb{Z}_p$.

Correct Ans: **B**

Itemcode : **HT1055**

Q55: Pick out the false statement.

A	$\frac{\mathbb{Q}}{\mathbb{Z}}$ is an infinite group each of whose element is of finite order.
B	$\frac{\mathbb{Q}}{\mathbb{Z}}$ has exactly one cyclic subgroup of order n for every n.
C	D_5 has 5 elements of order 2.
D	Number of conjugate classes for D_8 = 8.

Correct Ans: **D**

Itemcode : **HT1056**

Q56: Which one of the following statements is incorrect?

A	For $\lvert G\rvert$ = 200, there exists unique subgroup of order 25.
B	A group with $\lvert G\rvert$ = 15 is cyclic.
C	A group with $\lvert G\rvert$ = 1001 is simple.
D	A group G with order 108 is not simple.

Correct Ans: **C**

Itemcode : **HT1057**

Q57: Which of the following statements is incorrect?

A	Compact subset of every topological space is closed.
B	Sequentially compact metric space is totally bounded.
C	Discrete space with more than one point is disconnected.
D	If X is a metric space, S is a connected subset, $S \subseteq A \subseteq \bar{S}$ then A is connected.

Correct Ans: **A**

Itemcode : **HT1058**

Q58: $[a, b] \subseteq \mathbb{R}, f$ be a continuous function, then $f([a, b])$ is

A	Open and bounded.
B	Closed but not bounded.
C	Disconnected
D	Closed and bounded.

Correct Ans: **D**

Itemcode : **HT1059**

Q59: Which one of the following subsets of $\mathbb{R}^3$ is compact under usual topology?

A	$\{(x_1, x_2, x_3) : \ |x_i| < 1, 1 \le i \le 3\}$
B	$\{(x_1, x_2, x_3) : \ x_1 + x_2 + x_3 = 0\}$
C	$\{(x_1, x_2, x_3) : \ x_i \ge 0, 1 \le i \le 3\}$
D	$\{(x_1, x_2, x_3) : \ |x_i| \le 2, 1 \le i \le 3\}$

Correct Ans: **D**

Itemcode : **HT1060**

Q60: Let A be an $n \times n$ matrix over $\mathbb{R}$. Then which one of the following is false?

A	Every solution of the system $A^T AX = 0$ is also a solution of $AX = 0$.
B	The systems $AX = 0$ and $A^T AX = 0$ have the same set of solutions.
C	Rank $(A^T A)$ = Rank (A).
D	There exists $x_0 \in \mathbb{R}^n$ such that $Ax_0 = 0$ and $A^T A x_0 \neq 0$.

Correct Ans: **D**

Itemcode : **HT1061**

Q61: Let V be the set of all continuous functions on $[a, b]$. Then

A	V is not a vector space.
B	$\{\sin x, \sin^2 x, \sin^3 x\}$ is linearly independent in V.
C	$\{1, \cos x, \cos^2 x\}$ is linearly dependent in V.

D	*V* is a finite dimensional vector space.
Correct Ans: **B**	

Itemcode : **HT1062**

Q62: Let $A \in M_n(\mathbb{R})$ such that $A = A^T$. Then which one is incorrect?

A	Every eigenvalue of *A* is real.
B	If $x_1, x_2 \in \mathbb{R}^n$ are eigenvectors of A corresponding to distinct eigenvalues, then ${x_1}^T x_2 = 0$.
C	Every eigenvalue of *A* is positive.
D	There is an orthonormal set $\{x_1, x_2, \ldots, x_n\}$ in $\mathbb{R}^n$ such that $A\,x_i = \lambda_i x_i$, $i = 1,2,\ldots n$ for some $\lambda_i \in \mathbb{R}$.
Correct Ans: **C**	

Itemcode : **HT1063**

Q63: Let $A \in M_{m\times n}(\mathbb{C})$, $B \in M_{n\times m}(\mathbb{C})$. Then

A	BA and *AB* have same set of eigenvalues only if Rank (*AB*) = Rank (*BA*).
B	*AB* is diagonalizable iff *BA* is diagonalizable.
C	If $(x - \lambda)^r$ divides the characteristic polynomial of AB for some $\lambda \in \mathbb{C}, r \in \mathbb{N}$, then $(x - \lambda)^r$ divides the characteristic polynomial of BA.
D	If $(x - \lambda)^r$ divides the characteristic polynomial of BA for some $\lambda \in \mathbb{C}, \lambda \neq 0, r \in \mathbb{N}$, then $(x - \lambda)^r$ divides the characteristic polynomial of AB.
Correct Ans: **D**	

Itemcode : **HT1064**

Q64: If A is a 3 × 3 matrix having eigenvalues 0,2,3 with eigenvectors *u, v, w* respectively, then

A	The linear system *Ax* = *u* is consistent.
B	The linear system *Ax* = *v* + *w* is consistent.
C	The linear system *Ax* = *u* has infinitely many solutions.
D	The linear system *Ax* = *v* + *w* has a unique solution.
Correct Ans: **B**	

Itemcode : **HT1065**

Q65:

$$\text{If } A = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \text{ then}$$

A	|*A*| = -1.
B	*A* is diagonalizable over $\mathbb{R}$.
C	Minimal polynomial of *A* is $x^4 + x^3 + x^2 + x + 1$.
D	*A* has a rational eigenvalue.

Correct Ans: **C**

Itemcode : **HT1066**

Q66: If $f(z) = (e^z - i)^{-1}$, for all $z \in \mathbb{C}$ such that $e^z \neq i$, then which one of the following is correct?

A	f is entire.
B	All the poles of f lie on the imaginary axis.
C	f has a double pole.
D	All the poles of f lie on the real axis.

Correct Ans: **B**

Itemcode : **HT1067**

Q67: If $I_\gamma = \int_\gamma \frac{dz}{z}$, then

A	$I_\gamma = \log\sqrt{2} + i\frac{\pi}{4}$ if γ is the straight line from 1 to $1+i$.
B	$I_\gamma = 0$ if γ is the unit circle .
C	$I_\gamma = 2\pi i$ if γ is the circle with center $z = 3$ and radius 1.
D	$I_\gamma = 0$ for any closed curve γ in the complex plane.

Correct Ans: **A**

Itemcode : **HT1068**

Q68: The conformal map $w = z^2$ takes the half disk $\{z: |z| < 1, \text{Im } z > 0\}$ onto

A	$\{w: |w| < 1\}$
B	$\{w: \text{Re } w > 0\}$
C	$\{w: |w| < 1,\ 0 < \arg w \leq 2\pi\}$
D	$\{w: |w| < 1,\ 0 < \arg w < 2\pi\}$

Correct Ans: **D**

Itemcode : **HT1069**

Q69: $\int_\gamma (z^2 - 3|z| + \text{Im } z)\, dz$, where γ is the quarter circle centered at the origin and extending from 2 to 2i is

A	0
B	$\frac{28}{3} - \pi + \frac{38}{3}i$
C	$2\pi i$
D	$\frac{28}{3} - \pi - \frac{38}{3}i$

Correct Ans: **D**

Itemcode : **HT1070**

Q70: For the function $f(z) = \frac{1+z}{1-z^4}$, which one of the following is true?

A	All the singularities of f are poles.
B	f has a removable singularity.
C	f has an essential singularity.
D	All the poles of f are of order 2.

Correct Ans: **B**

Itemcode : **HT1071**

Q71: The map $f(z) = \frac{e^z-1}{e^z+1}$, maps the horizontal strip $S = \left\{z: \frac{-\pi}{2} < \text{Im } z < \frac{\pi}{2}\right\}$ conformaly onto

A	the right half plane.
B	the horizontal strip *S*.
C	the unit disk $\mathbb{D}$.
D	the first quadrant.

Correct Ans: **C**

Itemcode : **HT1072**

Q72: Let $S \subseteq \mathbb{R}$ such that $Int(S) \neq \emptyset$. Then which of the following is false?

A	*S* has a convergent sequence of real numbers.
B	*S* is uncountable.
C	*S* contains a closed interval as a proper subset.
D	*S* is a countable union of disjoint intervals.

Correct Ans: **D**

Itemcode : **HT1073**

Q73: If f and g are functions on $\mathbb{R}$, then which of the following is true?

A	If $f \; o \; g = g \; o \; f$, then $f = g$.
B	If $f \; o \; g = g \; o \; f$, then either f or g is an identity function.
C	If A and B are subsets of $\mathbb{R}$, then $f(A \cup B) = f(A) \cup f(B)$.
D	If A and B are subsets of $\mathbb{R}$, then $f(A \cap B) = f(A) \cap f(B)$.

Correct Ans: **C**

Itemcode : **HT1074**

Q74: Consider the sequence $a_n = (-1)^n \frac{n+1}{n}$, $n \geq 1$. Let $l = liminf \; a_n$ and $s = limsup \; a_n$. Choose the correct statement from below.

A	$l = s = 0.$
B	$-1 \le l < s \le 1.$
C	$\sum_{n=1}^{\infty} a_n$ is convergent.
D	$< a_n >$ is convergent.

Correct Ans: **B**

Itemcode : **HT1075**

Q75: Let $\mathbb{R}^2 = \{(x_1, x_2): x_1, x_2 \in \mathbb{R}\}$ and $<>: \mathbb{R}^2 \to \mathbb{R}$ be an inner product on $\mathbb{R}^2$ defined by

$$< x, y > = x_1y_1 + x_2y_2 \text{ for } x = (x_1, x_2), y = (y_1, y_2).$$

Then $\sup \{< x, y > + < y, z > + < z, x >: < x, x > = < y, y > = < z, z > = 1\}$ is

A	3
B	4
C	2
D	does not exist

Correct Ans: **A**

Itemcode : **HT1076**

Q76: Let *Y* be the set of all real-valued continuous functions defined on [0, 1] with supnorm. Let *X* be the subspace of all continuously differentiable functions in *Y*. Define $T: X \to Y$ by $T(x(t)) = x'(t).$ Then

A	T is not a linear transformation.
B	T is bounded.
C	Graph(T) is closed in $X \times Y$.
D	T is continuous.

Correct Ans: **C**

Itemcode : **HT1077**

Q77: Let *B*(*H*) be the set of all bounded linear transformations defined on a Hilbert space *H*, and *S* be the set of all self-adjoint operators in *B*(*H*). Then

A	*S* is a closed subset of *B*(*H*).
B	*S* is not a subspace of *B*(*H*) over $\mathbb{R}$.
C	*S* is an empty set.
D	*S* is neither open nor closed.

Correct Ans: **C**

Itemcode : **HT1078**

Q78: Let $\mathbb{Q}$ be the set of all rational numbers and *I* = [0, 1].

A	(a) Lebesgue outer measure of $I \cap \mathbb{Q}$ is ½.
B	(b) Lebesgue outer measure of $I \cap \mathbb{Q}^C$ is ½.
C	(c) Let *G* be the set of all numbers in [0, 1] which possesses decimal expansions not containing the digit 5 has measure 4/10.

D	(d) The set *G* defined in (c) has measure 0.

Correct Ans: **D**

Itemcode : **HT1079**

Q79: Let τ_1 be the standard topology on $\mathbb{R}$ and τ_2 be the cofinite toplogy on $\mathbb{R}$. Then

A	$(\mathbb{R}, \tau_1)$ is finer than $(\mathbb{R}, \tau_2)$.
B	$(\mathbb{R}, \tau_2)$ is finer than $(\mathbb{R}, \tau_1)$.
C	Both $(\mathbb{R}, \tau_1)$ is finer than $(\mathbb{R}, \tau_2)$. and $(\mathbb{R}, \tau_2)$ is finer than $(\mathbb{R}, \tau_1)$.
D	$(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$ are not comparable.

Correct Ans: **D**

Itemcode : **HT1080**

Q80: Let $(\mathbb{R}^2, \tau)$ be a topological space with standard topology τ and $A = \{(a, b) \in \mathbb{R}^2 \colon \sin a = 0 \text{ and } b \in \mathbb{Q}\}$. Then

A	$\mathbb{R}^2 - A$ is path-wise connected.
B	$\mathbb{R}^2 - A$ is disconnected.
C	If $B \subseteq \mathbb{R}^2$ is connected, then $\mathbb{R}^2 - B$ is path-wise connected.
D	A is a closed subset in $\mathbb{R}^2$.

Correct Ans: **A**

Itemcode : **HT1081**

Q81: To which district of H.P. did Jaiwant Ram, the first Speaker of H.P. Legislative Assembly, belong?

A	Bilaspur
B	Mandi
C	Chamba
D	Sirmour

Correct Ans: **C**

Itemcode : **HT1082**

Q82: Which Assembly constituencies of Bilaspur district of H.P. were initially represented by a husband-wife team during 1951-54 elections?

A	Ghumarwin and Gehrwin
B	Gehrwin and Kot Kehlur
C	Kot Kehlur and Bilaspur
D	Bilaspur and Ghumarwin

Correct Ans: **A**

Itemcode : **HT1083**

Q83: Which one of the following was a double-member constituency during the 1951-54 Vidhan Sabha elections in H.P.?

A	Theog
B	Rajgarh
C	Jubbal
D	Kasumpti

Correct Ans: **A**

Itemcode : **HT1084**

Q84: The inquiry Commission set-up in 2012 to look into all benami land deals in H.P. was headed by Justice ____________________.

A	V.K. Sharma
B	D.P. Sood
C	Sandeep Sharma
D	S.S. Negi

Correct Ans: **B**

Itemcode : **HT1085**

Q85: Approximately, what percentage of total geographical area of the country falls in H.P.?

A	1.2
B	1.7
C	2.3
D	2.5

Correct Ans: **B**

Itemcode : **HT1086**

Q86: What was the per capita income in H.P. (in rupees at current prices) during 1950-51?

A	160
B	180
C	240
D	290

Correct Ans: **C**

Itemcode : **HT1087**

Q87: When was Rajiv Gandhi Ann Yojna introduced in H.P.?

A	2012
B	2013
C	2014
D	2015

Correct Ans: **B**

Itemcode : **HT1088**

Q88: Which district of H.P. is the main beneficiary of Shah canal?

A	Bilaspur

B	Hamirpur
C	Kangra
D	Una

Correct Ans: **C**

Itemcode : **HT1089**

Q89: How many districts of H.P. are excluded from Mid-Himalayan Watershed Project?

A	One
B	Two
C	Three
D	Four

Correct Ans: **B**

Itemcode : **HT1090**

Q90: To which district of H.P. does Charanjit Singh, who was captain of Indian Hockey Team during the 1964 Tokyo Olympics, belong?

A	Bilaspur
B	Una
C	Kangra
D	Chamba

Correct Ans: **B**

Itemcode : **HT1091**

Q91: Sania Mirza is associated with ______________________.

A	Badminton
B	Hockey
C	Lawn Tennis
D	Soft Ball

Correct Ans: **C**

Itemcode : **HT1092**

Q92: Where is the Headquarters of Southern Command of Indian Army?

A	Secundrabad
B	Pune
C	Kochi
D	Vizag

Correct Ans: **B**

Itemcode : **HT1093**

Q93: What is the upper age limit for eligibility to Accident Insurance under Pradhan Mantri Suraksha Bima Yojna?

A	60 years
B	65 years

C	70 years
D	75 years

Correct Ans: **C**

Itemcode : **HT1094**

Q94: Which one of the following is NOT included in Tertiary sector of Indian economy?

A	Animal Husbandry
B	Banking
C	Tourism
D	Insurance

Correct Ans: **A**

Itemcode : **HT1095**

Q95: Who is the author of Geet Gobind?

A	Jai Dev
B	Patanjli
C	Asvaghosh
D	Kali Das

Correct Ans: **A**

Itemcode : **HT1096**

Q96: What is the currency of Iraq?

A	Yen
B	Dinar
C	Rial
D	Dirham

Correct Ans: **B**

Itemcode : **HT1097**

Q97: Who propounded the Human Development Index (HDI)?

A	Ranjit Guha
B	Amartya Sen
C	Mahbub-ul-Haq
D	Urjit Patel

Correct Ans: **C**

Itemcode : **HT1098**

Q98: Among the following, who was given Nobel Prize for Peace in 2014?

A	V.S. Naipaul
B	V. Ramakrishnan
C	Kailash Satyarthi
D	None of these

Correct Ans: **C**	

Itemcode : **HT1099**

Q99: Suez canal connects Mediterranean Sea and ________.

A	Baltic Sea
B	Red Sea
C	Caspian Sea
D	Dead Sea

Correct Ans: **B**

Itemcode : **HT1100**

Q100: Given below are the names of some countries and their capitals. Find the mis-match.

A	Angola - Luanda
B	Egypt - Cairo
C	Uganda - Ankara
D	Sudan - Khartoum

Correct Ans: **C**